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THE GRAPHIC METHOD FOR THE STUDY OF RECIPROC ENVELOPED SURFACES WITH POINT OF CONTACT I – THE HOBBING CUTTER – ALGORITHM –

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ABSTRACT

The paper shows a new graphic method, developed as a specific application in AutoLISP programming, for the study of reciprocal enveloped surfaces. There are two steps for the resolution by this graphic method: 1) the elaboration of „solid" model of the rack-tool; 2) it is established the primary-peripheral surface of the snail-tool.

1. Introduction

The problem of the generation of surfaces with point of contact is a matter if generation by winding of a family of surfaces depending on two parameters.

In the general wording OLIVIER, [1], for the surface Σ

$$\Sigma : F(xyz) = 0 \tag{1}$$

which describes a family depending on two independent parameters, be it α and β, the family's envelope

$$((\Sigma)_\alpha)_\beta : F(xyz,\alpha,\beta) = 0 \tag{2}$$

is established by eliminating the two parameters from the system of equations:

$$F(xyz,\alpha,\beta) = 0;\quad F'_\alpha = 0;\quad F'_\beta = 0. \tag{3}$$

The application of the second theorem OLIVIER, [1], in most cases is difficult, the analytic equations being extremely complicated.

There are known and used other methods, too, developed in the basis of the theorems of the method "minimum distance" or of the family of "substitutant" circles [3].

In the following lines, is presented a graphic method, developed as a specific application in the AutoLISP programming medium, on the basis of an original soft dedicated to such a problem.

2. Profiling algorithm

In figure 1, are represented the reference systems facing which are defined the wire of cylindrical surfaces to be generated, as well as the future primary-peripheral surface of the snail-tool:

- **xyz** - is the fixed system of reference, having the axis **z** superposed to the axis of rotation of the semi-product;
- $x_0y_0z_0$ - is the fixed system of reference, having the axis y_0 superposed to the axis of rotation of the snail-tool;
- **XYZ**–mobile system, solidary to the surface to be generated (of the solid, limited by the surface to be generated);
- ξηζ–mobile system, solidary to the intermediary surface (the surface of the rack's flank – the solid representing the rack conjugated of profile **S**);
- $X_1Y_1Z_1$ – mobile system, solidary to the primary-peripheral surface of the snail-tool (the solid of the snail-tool).

The resolution by „ the graphic method „ of the problem of determination the primary-peripheral surface of the snail-tool, requires the running of two stages:

a) determination of the primary peripheral surface of the rack-tool and the elaborations of its „ solid „ model;

b) starting from the rack's „ solid „ model, it is established the primary-peripheral surface of the snail-tool (the „ solid „ model „ of the snail-tool).

Obviously, there can be realised plane intersections or with helical surfaces of the primary-peripheral surface of the snail-tool, on the purpose of establishing the theoretical form of the snail-tool's cutting-edge.

The establishment of the "solid model" of the rack-tool, reciproc enveloped to the surface to be generate, (the surface belonging to an ordinate whire of surfaces and associated to a circular centroid), was presented in [4], [5].

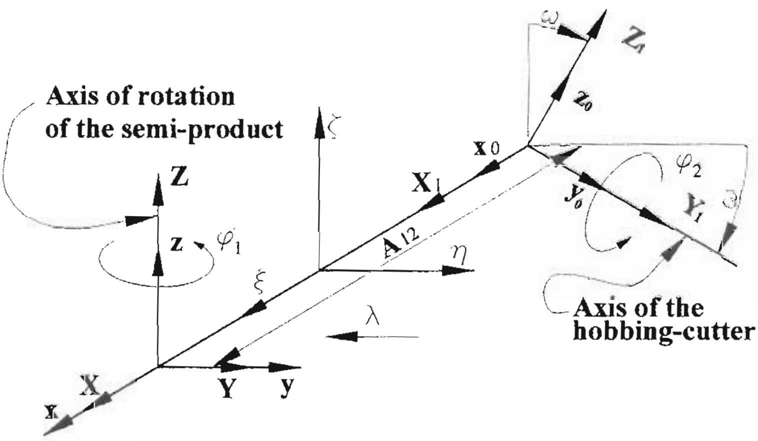

Fig.1. Reference systems of the semi-product and of the hobbing-cutter.

2.1. The cinematic of the process of generation

The process of generation to the working with a snail-tool, requires the achievement of the movements:

- rotation of the semi-product and, in the same time, of the surface's (profile's) whire **Σ** which limits it (rotation of axis **Z** and angular parameter φ_1);
- rotation of the snail-tool, of axis $\mathbf{Y_1}$ and angular parameter φ_2.

In solving the problem of generation, it can be used an auxiliary surface (the intermediary surface), which, performing an imposed movement – translation of parameter **λ** – following the axis "η" of the system of reference ξηζ, „carries out" simultaneously the conditions between the parameters of movements:

$$\lambda = R_r \cdot \varphi_1; \qquad (4)$$

$$\lambda = p \cdot \varphi_2 \cdot \cos\omega , \qquad (5)$$

where $\mathbf{R_r}$ is the thread-ray of the profile's whire **Σ**; **p** – helical parameter of the snail representing the helical tool's flank; **ω** – inclination angle of the snail-tool's axis.

The relations (4) and (5) are arbitrary relations, their form depending on the kinematic possibilities of a machine-tool to make gear-wheels with the snail-tool (the PHAUTER machine).

The movement of rotation of the system **XYZ** is described by a transformation such as:

$$x = \omega_3^T(\varphi_1) \cdot X \qquad (6)$$

The translation of the system ξηζ, (the imposed movement of the auxiliary surface) is given by:

$$x = \xi + a \qquad (7)$$

$$\text{with} \quad a = \begin{Vmatrix} -R_r & -R_r \cdot \varphi_1 & 0 \end{Vmatrix}. \qquad (8)$$

The rotation of the system $\mathbf{X_1Y_1Z_1}$, of axis $\mathbf{Y_1}$ and parameter ω_2, is given by

$$x_0 = \omega_2^T(\varphi_2) \cdot X_1, \qquad (9)$$

which, reported to the system **xyz**, by the transformation of coordinates

$$x_0 = \beta(x - b) \qquad (10)$$

$$\text{with } \beta = \begin{Vmatrix} 1 & 0 & 0 \\ 0 & \cos\omega & -\sin\omega \\ 0 & \sin\omega & \cos\omega \end{Vmatrix} \text{ and } b = \begin{Vmatrix} -A_{12} \\ 0 \\ 0 \end{Vmatrix}$$

becomes

$$x = \beta^T \cdot \omega_2^T(\varphi_2) \cdot X_1 + b \qquad (11)$$

The assembly of absolute movements (6) + (11) permits the establishment of the relative movements between the semi-product **Σ** and the intermediary surface **I**

$$\xi = \omega_3^T(\varphi_1) \cdot X - a \qquad (12)$$

as well as the movement of the intermediary surface face to the system of the snail-tool,

$$X_1 = \omega_2(\varphi_2) \cdot \beta \cdot [\xi + a - b]. \qquad (13)$$

2.2 The intermediary surface – I

Knowing that, the form of the profile (see figure 2a) (of the surface) to be generated – Σ, in principle by the form:

$$\Sigma: \; X = X(u); \quad Y = Y(u), \qquad (14)$$

with **u** – the variable parameter, in the relative movement (12), envelopes the profile of the **intermediary surface I**, figure 2b, the numerical identification of the profile **I** is made by appealing to an identification algorithm, whose logical scheme is presented in the figure 3.

__Note__: The decrease of the incremental step of the parameter "φ_1" may lead to an accuracy of identification big enough for the necessities of projecting tools to generate by winding the profiles.

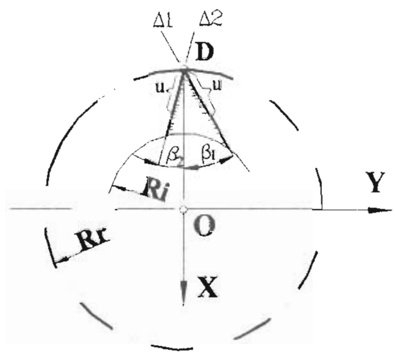

Fig.2a. The form of the profile to be generated

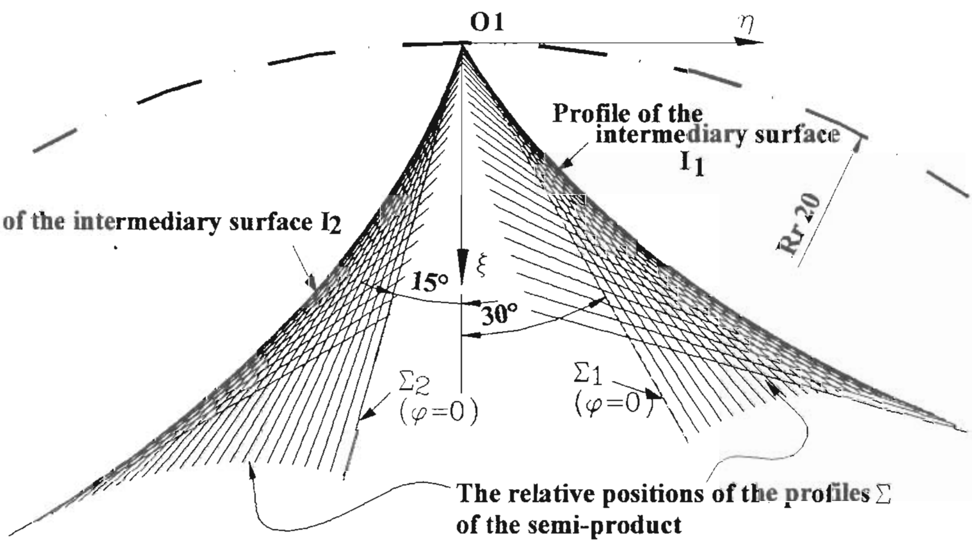

Fig. 2b. Profiles of the intermediary surface (the rack), envelope of the relative positions of the profile Σ of the semi-product

2.3. Surface – S – primary periphal surface of the snail-tool

Knowing the profile **I** of the intermediate surface (in all curent situations this being a cylindrical surface), can be modelled a "solid" limited by this intermediate surface, see figure 4 also.

Based on the solid model of the rack – I_S – and of the kinematics given by (13), for a point belonging to the solid, conveniently chosen, be it $\mathbf{M(\xi_M,\eta_M,\zeta_M)}$, (model $\mathbf{I_S}$), it is put on position for a small increment of the angular parameter φ_2, face to system of reference $X_1Y_1Z_1$.

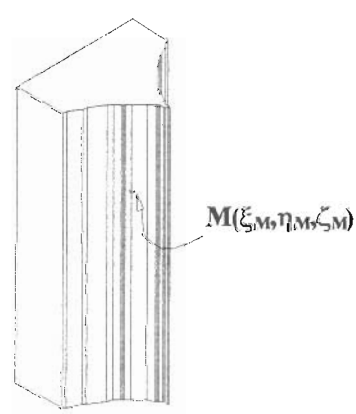

Fig. 4. "S

surface **I** – numericaly identified – $\mathbf{I_S}$

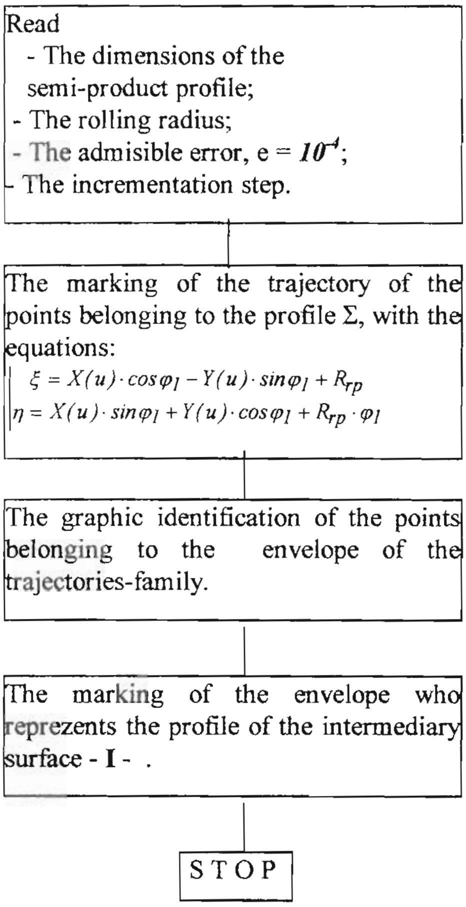

Fig. 3. Algorithm for the numerical "identification" of the profile

The assembly of these relative "positions" of the solid model - I_S- face to the system $X_1Y_1Z_1$ is "decreased" from the solid model, initially cylinder of revolution, of the snail-tool. It is used the command "SUBTRACT" from AutoCAD. Command used for modeling in the three-dimensional space. The result is the "solid model" of the snail-tool – S_S.

2.4.Plane and helical sections of the "solid model" of the snail-tool - S_S -

The effective cutting-edge of the snail-tool can be established by the intersection of the surface's solid model – S_S – with the surface representing the helical channel of the cavity between the tool's teeth, figure 5.

In figure 5, are represented the relative position of the disk-tool for the working of the helical channel of the snail-tool's teeth face to that – solid S_S – as well as the profile of the tool (solid SD_S).

The axial section of the peripheral primary surface of the snail-tool is defined in the system $X_1Y_1Z_1$, and the normal section on the propeller of the helical channel of the tool's teeth, is defined in the system $X_2Y_2Z_2$.

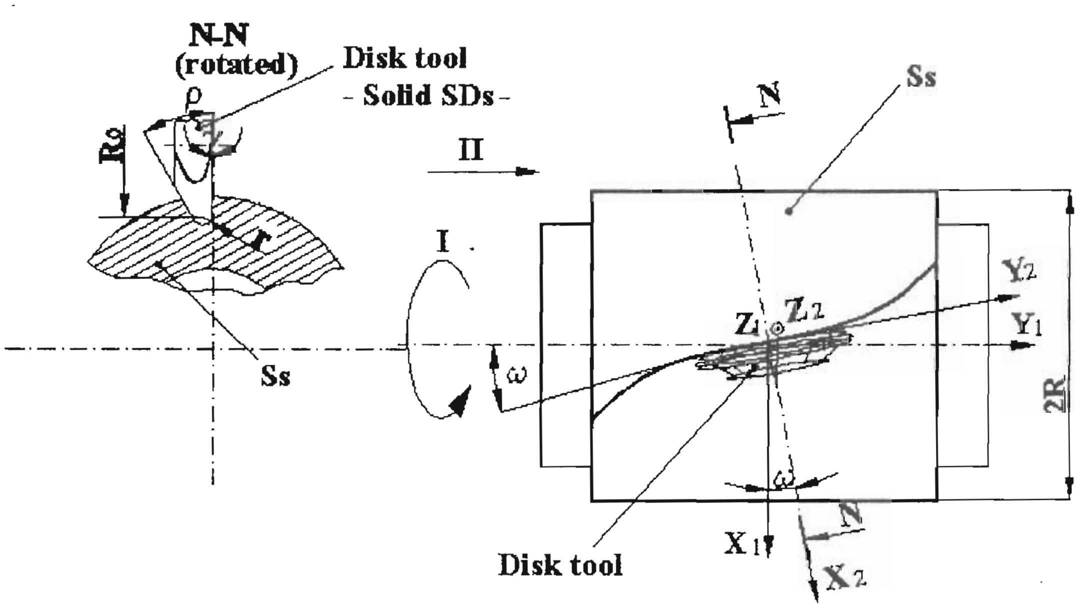

Fig. 5. Sections of the solid – S_S – of the snail-tool with the helical channel of the teeth.

The profile resulted from the intersection of the solids S_S and SD_S represents the cutting-edge's profile in an approximate form it can be accepted the substitution of the cutting-edge, thus defined, for the edge resulted from the intersection of the solid S_S with a normal plane on the propeller of the snail-tool's teeth (the plane Y_2Z_2) – the normal section on the propeller.

Also, it can be established, for technological reasons, connected to the measurement of the snail-tool, the profile of the surface **S** in an axial plane of this tool, figure 6.

The problem is solved by the intersection of the solid – S_S – with the plane P_A (the plane X_1Y_1) – the plane, using the known command of the AutoCAD programme.

Obviously, in both cases, the two sections, see figure 7, will be numerically identified.

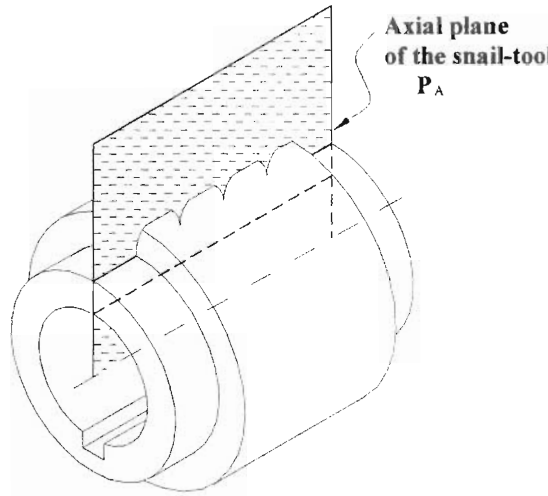

Fig. 6. Axial section of the snail-tool

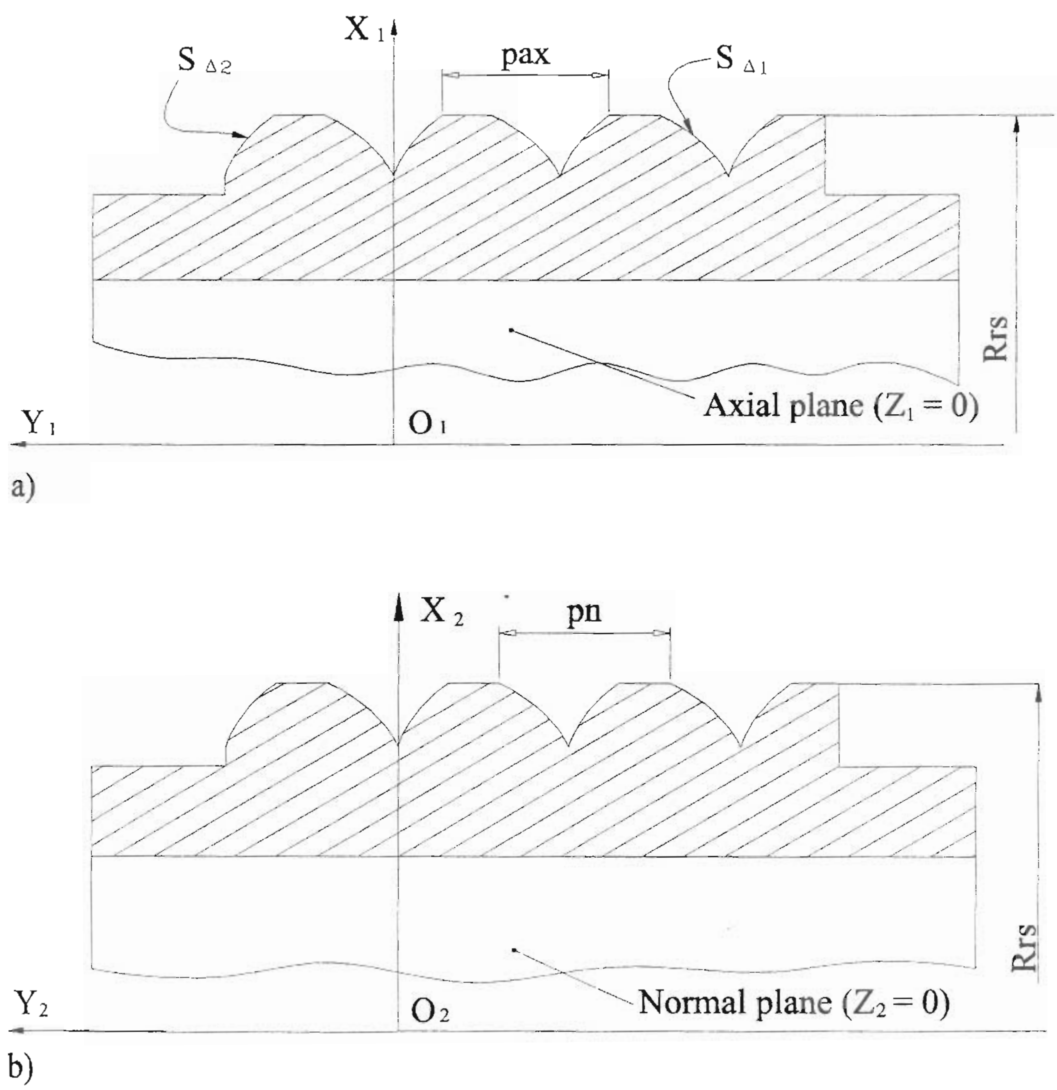

Fig. 7. Sections axial (a) and normal (b) of the solid S_S. ($S_{\Delta 1}$,$S_{\Delta 2}$- flanks of the snail-tool corresponding to the flanks **Δ1** and **Δ2** of the semi-product (Fig. 2a)).

3. Application to the proposed method. The snail-tool for a square shaft.

It proposes, as follows, an application to the proposed method – the snail-tool used to generate a shaft with a square section.

In the figure 8, it is presented the profiles whire who limit the square shaft, in connection with the figure 1.

We define the equations of the surface **Σ** of the whire:

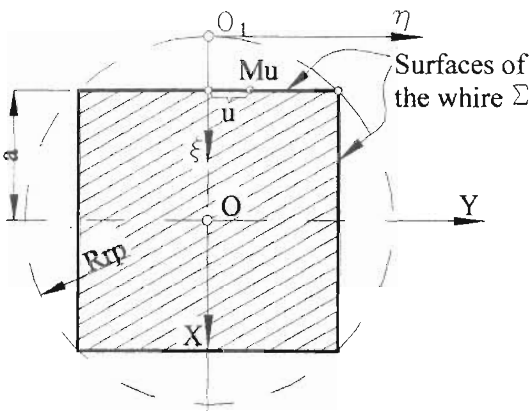

Fig. 8. Surfaces of the whire Σ

$$\Sigma: \; X = -a; \; Y = u, \qquad (15)$$

with **u** variable parameter.

On the basis of the movement (8) and of the algorithm presented in the chapter 2.2. , is established the profile of the intermediary surface for a square shaft with the side **b = 2 · a ; a =** ***10*** **mm**, see figure 9, also, subsequent numerically identified, on the base of the algorithm presented in figure 3, table 1.

Table 1

The crossing profile of the intermediary surface	
ξ [mm]	η [mm]
0.000284	-11.1072
1.174541	-9.72167
2.315515	-7.84951
3.279748	-5.51437
3.909969	-2.90225
4.142605	0
3.909969	2.90225
3.279748	5.514369
2.315515	7.849505
1.174541	9.721674
0.000284	11.10721

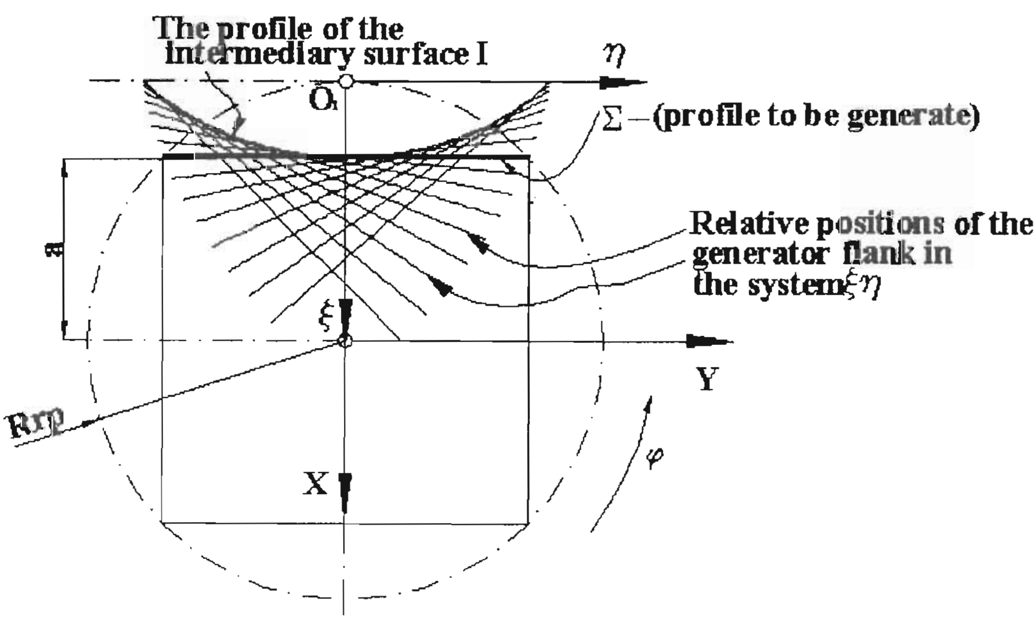

Fig.9a. The rack profile for a square shaft

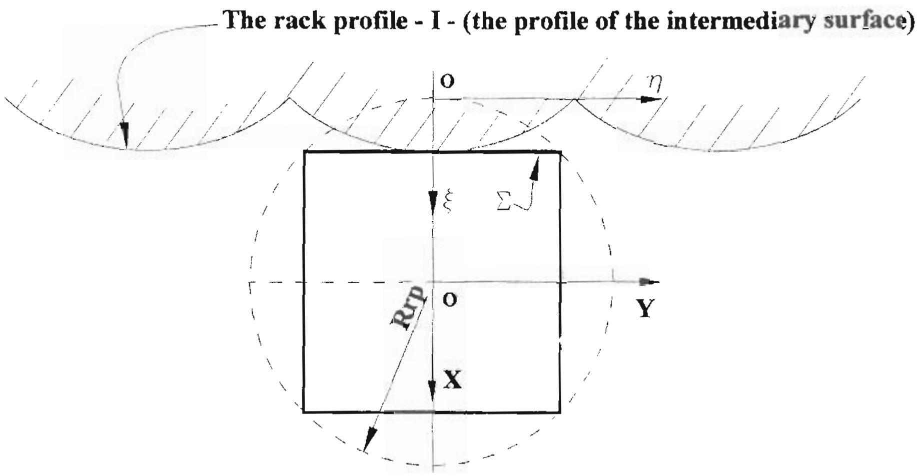

Fig. 9b. The intermediary surface profile

In figure 10, are presented the successive positions of the solid $\mathbf{I_S}$ – limited by the surface **I**, in the movement (3) face to the system of reference of the snail-tool.

As a result of a "decrease" (using initially, revolution cylinder with the ray $\mathbf{R_{rs} = 50}$ **mm**, of the successive solids $\mathbf{I_S}$ results, figure 11, the solid of the snail-tool $\mathbf{S_S}$.

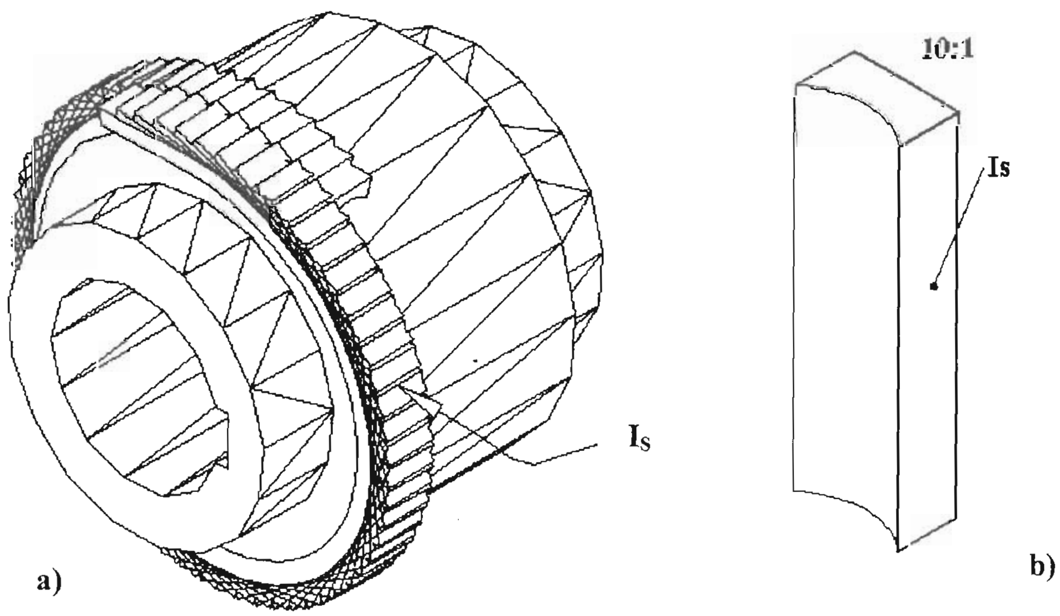

Fig. 10. Relative positions of the solid $\mathbf{I_S}$ towards the system of the snail-tool (a) , the intermediary surface's solid (b) .

The angle of bank of the snail's screw, is given by the relations:

$$tg\omega = \frac{p_c}{2 \cdot \pi \cdot R_{rs}} \quad \text{where} \tag{16}$$

p_c is the pitch-slots, $p_C = \frac{2 \cdot \pi \cdot R_{rp}}{z_C}$, (17)

For the concrete case, **$z_c = 4$**, **$R_{rs} = 50$ mm**. Results **$\omega = 0{,}0705932$ rad**.

In the figure 11 is represented the solid S_S – the primary snail of the snail-tool.

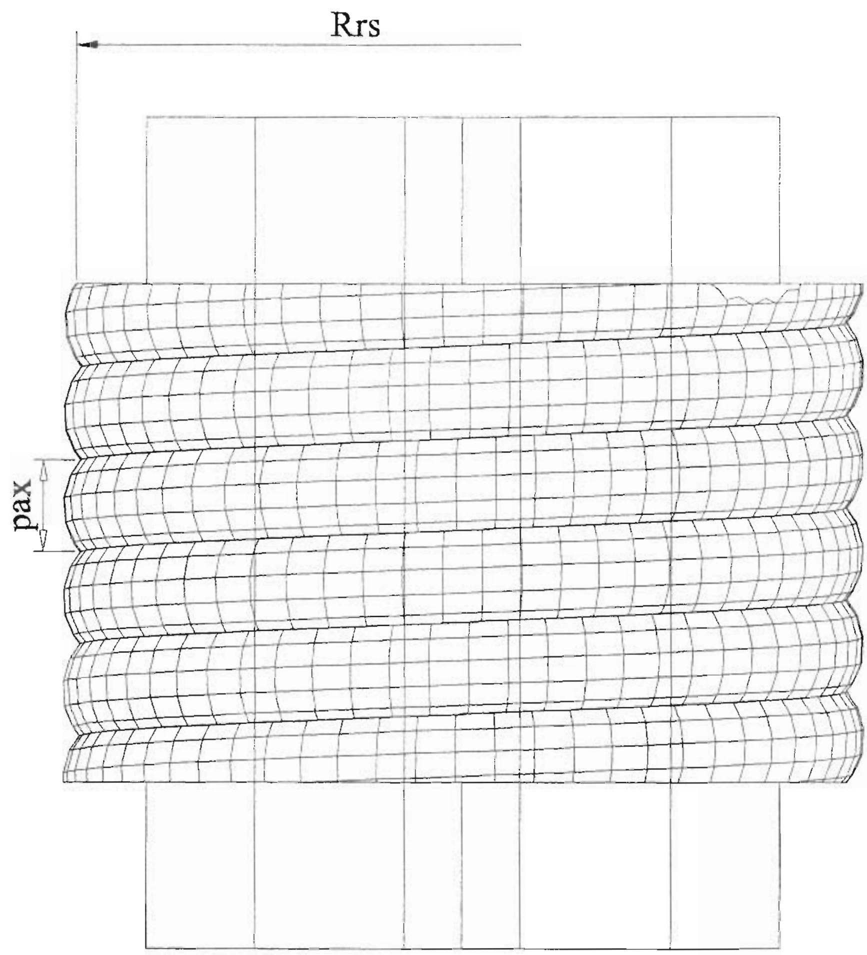

Fig. 11. The solid S_S – the primary snail of the snail-tool.

Table 2

Axial section	
X_I [mm]	**Y_I [mm]**
50.042516	-11.0900
50.106171	-11.0400
50.190842	-10.9900
50.254361	-10.9400
50.401931	-10.8900
:	:
:	:
50.401931	10.8900
50.254361	10.9400
50.190842	10.9900
50.106171	11.0400
50.042516	11.0900

Table 3

Normal section	
X_M [mm]	**Y_M [mm]**
49.9178601	-11.062
49.9813565	-11.012
50.0658166	-10.962
50.1291774	-10.912
50.2763798	-10.862
:	:
:	:
50.2763798	10.862
50.1291774	10.912
50.0658166	10.962
49.9813565	11.012
49.9178601	11.062

Table 4

Contact line $L_{I,S}$		
X_I [mm]	**Y_I [mm]**	**Z_I [mm]**
50.2542	-10.8696	-0.0167
50.4018	-10.7069	-0.0253
50.6249	-10.4484	-0.0371
50.9265	-10.0733	-0.0506
51.5303	-9.2227	-0.0707
:	:	:
54.1421	0	0
:	:	:
51.5303	9.2227	0.0707
50.9265	10.0733	0.0506
50.6249	10.4484	0.0371
50.4018	10.7069	0.0253
50.2542	10.8696	0.0167

Axial and normal sections of the solid S_S – see chapter 2.4, are presented in the figure 12 and 13, and the "identified" coordinates, in table 3 and 4.

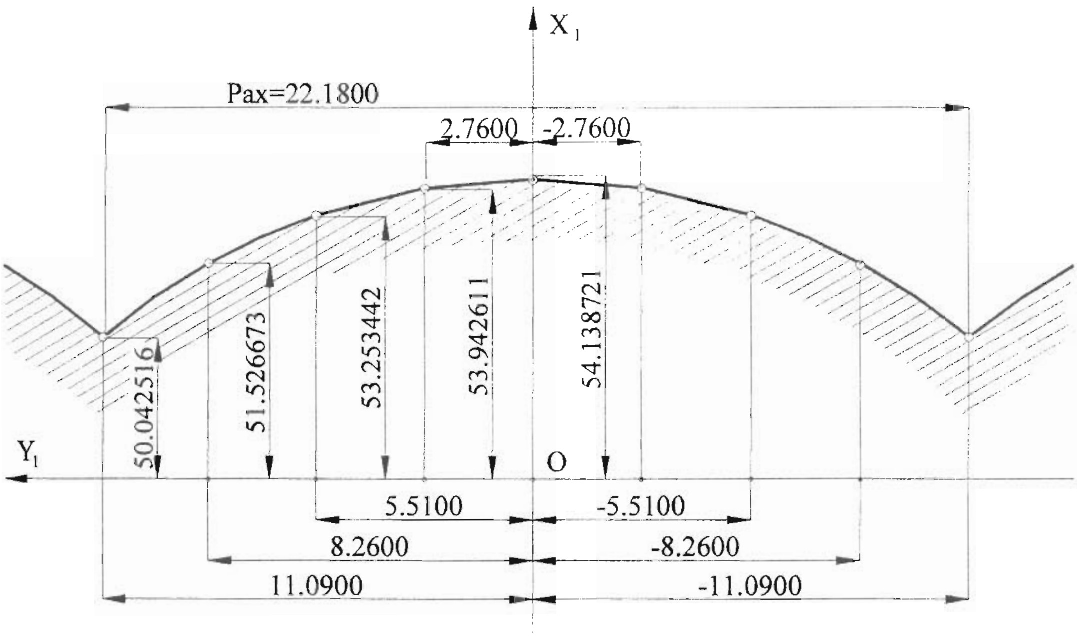

Fig. 12. The axial section of a snail-tool for the working of a square shaft.

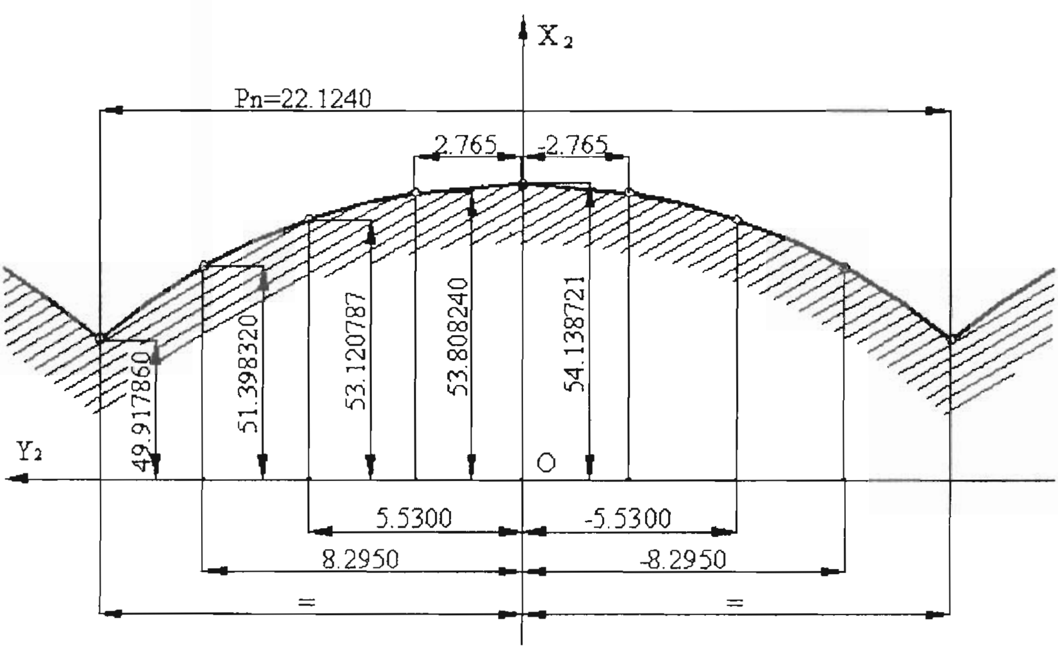

Fig. 13. The normal section of the snail-tool for a square shaft

The contact line

It is defined the contact line of the primary peripheral surface of the snail-tool with the intermediary surface, as being their tangent line for a relative position (be it $\varphi_2 = 0$, from (13).

In the figure 14, are represented, on the **rack's solid** (the intermediary surface), the contact lines between the semi-product Σ and the intermediary surface (the rack) – $L_{\Sigma,I}$, as well as the contact line between the surface **I** and the solid **S** of the snail-tool – $L_{I,S}$.

The intersection of the two contact lines, establishes the contact point of the surfaces Σ and S (see figure 14 and table 4).

Note:

The contact line $L_{I,S}$ is a rectilinear generator of the cylindrical surface I (the rack).

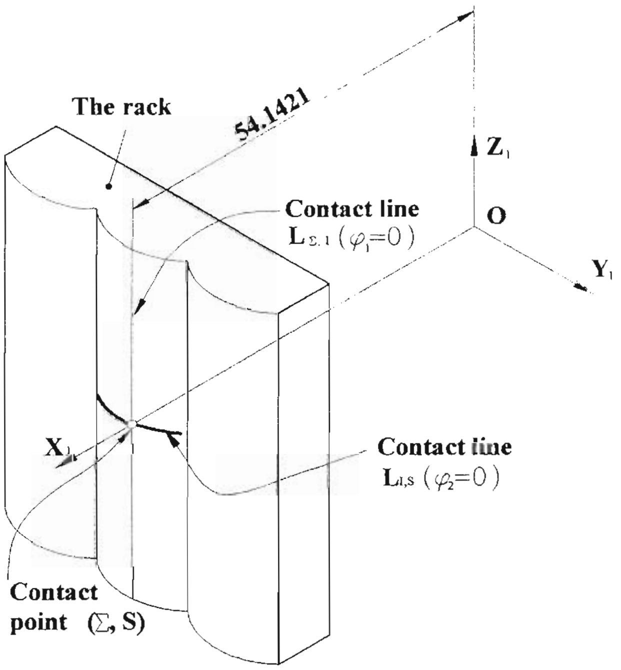

Fig.14. The rack's solid. Contact lines on the rack's solid.

4. Conclusions

The graphic method presented represents a viable alternative of the analytic method and is easy to apply even by the users with little knowledge in surfaces winding area.

The method is precise, and most of all eloquent by its way of expression.

The programming medium AutoLISP, by the facilities it offers, favors the easy obtain of some constructive technological elements of the snail-tool, difficult to obtain by the classic analytic methods.

The original programmes, developed on the AutoLISP programming medium ensure important facilities in the algorithm's application for the surfaces of any kind.

5. References

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METODĂ GRAFICĂ PENTRU STUDIUL SUPRAFEŢELOR RECIPROC ÎNFĂŞURĂTOARE CU CONTACT PUNCTIFORM
I– FREZA MELC- ALGORITM
(Rezumat)

Lucrarea prezintă o nouă metodă grafică dezvoltată în mediul de programare AutoLISP, pentru studiul suparfeţelor reciproc înfăşurătoare. Sunt două etape în rezolvarea acestei probleme:

a) determinarea suprafeţei periferice primare a sculei cremalieră şi elaborarea modelului solid al acestei scule;

b) pornind de la modelul solid al sculei cremalieră este determinată suprafaţa periferică primară a sculei melc (modelul solid al sculei melc).

LA MÉTHODE GRAPHIQUE POUR L'ÉTUDE DES SURFACES ENVELOPPÉES PAR RECIPROC AVEC LE POINT DE CONTACT
I – LE COUPEUR DE FRAISAGE - ALGORITHME
(Résume)

Le papier montre une nouvelle méthode graphique, développée comme application spécifique dans AutoLISP programmant, pour l'étude des surfaces enveloppées réciproques. Il y a deux étapes pour la résolution par cette graphique méthode :

1) l'élaboration du modèle "plein" de l'armoire outil;

2) on l'établit la surface primaire-périphérique de l'escargot outil.